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# Fractals: An Exposition

Fractals are a relatively new concept in geometry. Most concepts for Euclidean geometry, the division of geometry which deals with lines, circles, triangles, and other standard shapes, stem from the Late Greek and Early Roman times. Considering the age of mathematics, the study of fractals is new because it dates to the beginning of this century. However, the age of computers brought about an explosion into this yet untamed universe of math. Before this innovation, mathematicians could only visualize what they were discussing. But now, fractals are the mathematician's answer to chaos and therefore can be used to help scientists better understand nature and the universe. Scientists can define any structure from a snowflake to a mountain or even an entire planet with this new division in Mathematics. Thus, fractals define our universe.

Benoit B. Mandelbrot is a key figure behind the rise of this new science. A Professor of mathematical Sciences at Yale and an IBM Fellow, Mandelbrot is the man who coined the term "fractal" in 1975. Mathematicians, such as Gaston Julia, only defined them as sets before this and could only give properties of these sets. Also, there was no way for these early fractal researchers to see what they were hypothesizing about. As Mandelbrot states in *The Fractal Geometry of Nature,* "I coined *fractal* from the Latin adjective *fractus*. The corresponding Latin verb *frangere* means ‘to break...’" (4). Mandelbrot used this particular root because of how he defines fractals. In contrast, Euclidean geometry, which has its figures in a particular dimension (e.g. a square is two-dimensional), fractals have fractional dimensions. They do not exist in just one dimension but can encompass part of another. Therefore, fractals can represent everyday objects such as the sponge mentioned in *The Heart of Mathematics* and many other natural objects which Euclidean geometry can only imitate by glossing over the irregularities. This glossing over occurs because the whole number dimensions of Euclidean geometry, such as the first or third dimensions, are used constantly in our modern world. Defined geometrically, a zero-dimensional figure would just be a point. It has no height, width, or length. A one-dimensional figure is characterized by a line, which has only length. In two-dimensions, a figure would only have length and width, and would be on a plane. In three-dimensions, a figure, like a box, has all three components of length, width, and height. Conversely, fractals explore the spaces in between these dimensions, figures which do not fit so nicely into these categories as planes, lines, or boxes do.

Thus, not only is the previously described step into the fractional dimensions of fractals important, but also the fact that most fractals are self-similar and infinitely complex. The latter is defined as a picture that has no end to the magnification of itself. It is possible to pick a point and zoom in infinitely many times and still end up with some sort of shape. One will never come to a single point, line, plane, or surface that defines the fractal. The former means that the fractal is made of smaller versions of the larger pictures. Because the fractal is infinitely complex, self-similarity also goes to infinity.

The Sierpinski triangle, named after the Polish mathematician Waclaw Sierpinski, is an excellent example of the infinite complexity and self-similarity of fractals. Moreover, it is the composition of a series of triangles shrunk and moved repeatedly. This is done by reducing an image by one-half, copying the image three times, and placing the images in an equilateral triangle form. In like fashion, the steps are then repeated with the new figure. As the process is repeated infinitely many times, a Sierpinski triangle starts to form. This can be done with any image –even a Frisbee disc. Magnifying any portion of the triangle will produce another view of the same triangle. For these reasons, the Sierpinski Triangle is both self-similar and infinitely complex.

The mathematics behind fractals is quite simple considering the fact that these numbers represent chaos, something that we previously thought indefinable and indiscernibly complex. The process deals basically with sets and imaginary numbers. Sets are an easy concept to understand and will be left undefined in this essay. However, a brief explanation of imaginary numbers is necessary.

Imaginary numbers are just that, imaginary. Furthermore, they do not have any physical representation. Mathematically, they are represented by i, where is defined as. Numerically, imaginary numbers are undefined because there is no number that can be squared and equal -1. In addition, these numbers are usually written in the form of a+bi, where a and b are real numbers. Graphically, they are represented in the same coordinate plane as regular numbers; however, the y-axis is now the imaginary axis. The point is represented as the ordered pair (a,bi).

The way a computer knows how to draw fractals is by taking an equation entered in this form and seeing at which points the answer tends to infinity and how many iterations, or repetitions, it takes for this to occur. To specify, the iterations consist of putting a number into the equation and either getting an output of a number or an output that tends to infinity. If an answer is received that is a number, then that number is put into the equation until the output tends to infinity. Then the computer assigns certain colors to the number of times it takes for the output to go towards infinity. For example, say one iteration causes the computer to make the point blue, two iterations to make the computer color the point red, and so on until the computer reaches a preset limit. A Julia set is the place where all the chaotic behavior of a complex function occurs. Furthermore, the point where the output values that go to infinity meet the output values that do not is the interesting part of the fractal.

The Mandelbrot set is the most interesting fractal discovered to date. First seen in 1980 by Benoit B. Mandelbrot, it has been found to hold every connected Julia set inside of its picture. Heinz-Otto Peitgen, states of the Mandelbrot set, in *The Science of Fractal Images*: "[it is] the pictorial manifestation of order in the infinite variety of Julia sets" (177). In a connected Julia set if a fractal is connected, it consists of a succession of lines. This means that the points in the Mandelbrot set are all tied together with no single points off on their own. The different sets can be found by zooming into certain regions on the picture. Mandelbrot's fractal is also infinitely complex while being self-similar.

Fractals go beyond the pure mathematics of the concept as the practical uses are just starting to be found. In being able to identify natural structures with mathematical formulas, we can predict and hypothesize about the future of our environment, species, or many other natural events. We could predict volcanic eruptions or the way the stock market behaves. Fractals are all around us, from the weather, to a pile of fallen leaves, to the human body. With the knowledge that we are surrounded by these shapes we can make many predictions about our planet or universe. The use of fractals can help us predict the weather, predict the way a rock will fracture, and represent nearly limitless other possibilities.

There are many objects in our universe that behave as fractals. The most interesting topic is that of the fractals present within our own body. As Homer Smith, a computer engineer once observed, "If you like fractals, it is because you are made of them. If you can't stand fractals, it's because you can't stand yourself". This statement is true in our anatomy because of the cascade of ever smaller blood vessels feeding the heart. Even on the folds on the surface of the brain, one can see the pattern of "self-similar scaling". Even the very beating of the heart is shown to be chaotic. Only when the heart becomes regular is there a problem; the patient has a heart attack. The heart and other bodily functions thrive on irregularity. Thus, chaos reigns throughout the body. With our knowledge of fractals, we can perhaps learn more about ourselves.

Fractals permeate our lives. The very planet we stand on is one huge fractal waiting to be discovered. Scientific discoveries make our world smaller but few tend to bring the universe into our grasp while pushing it out of our reach at the same time. Although we are probing into the deepest secrets the universe holds, as with fractals of infinite complexity, we will never reach the end. From the smallest amoebae to the outermost galaxy, we have only begun to define our universe.

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