Group:

Present:

1. Cycloid - the path of a point on the perimeter of a circle rolling along a plane. General equation:  $x = a(t - \sin(t)), y = a(1 - \cos(t))$ , where a is the radius of the rolling circle.

With your calculator in RADIAN and PARAMETRIC mode, graph  $X_{1T} = T - \sin(T), Y_{1T} = 1 - \cos(T)$ , using the window  $T \in [0, 2\pi, \frac{\pi}{24}], X \in [0, 13, 1], Y \in [0, 4, 1]$ . Use ZSQUARE after the graph is sketched. The curve shown is the path traced by a point on a rolling circle. Change TMAX to  $4\pi$  to see the path for two revolutions of the circle.

Sketch what you see here:

## Spirograph curves

2. Hypocycloid - the path of a point on the perimeter of a circle which is rolling around the inside of a fixed circle. General equation:  $x = (a - b)\cos(t) + b\cos(\frac{a-b}{b}t), y = (a - b)\sin(t) - b\sin(\frac{a-b}{b}t)$ , where a is the radius of the fixed circle, and b is the radius of the rolling circle.

Graph  $X_{1T} = 1.5 \cos(T) + 0.5 \cos(3T), Y_{1T} = 1.5 \sin(T) - 0.5 \sin(3T)$  for  $T \in [0, 2\pi, \frac{\pi}{24}]$  in the ZDECIMAL window. (Here a = 2, b = 0.5)

Next graph  $X_{1T} = \cos(T) + 1.5 \cos(\frac{2T}{3}), Y_{1T} = \sin(T) - 1.5 \sin(\frac{2T}{3})$  for  $T \in [0, 6\pi, \frac{\pi}{12}]$ . (Here a = 2, b = 1.5, and the small circle needs to go around the inside of the fixed circle three times to close the curve.)

Sketch both curves here:

3. Epicycloid - the path of a point on the perimeter of a circle which is rolling around the outside of a fixed circle. General equation:  $x = (a + b)\cos(t) - b\cos(\frac{a+b}{b}t), y = (a+b)\sin(t) - b\sin(\frac{a+b}{b}t)$ , where a is the radius of the fixed circle, and b is the radius of the rolling circle.

Graph  $X_{1T} = 2\cos(T) - 0.5\cos(4T), Y_{1T} = 2\sin(T) - 0.5\sin(4T)$  for  $T \in [0, 2\pi, \frac{\pi}{24}]$ in the ZDECIMAL window. (Here a = 1.5, b = 0.5)

Next graph  $X_{1T} = 2.2 \cos(T) - 0.8 \cos(\frac{11T}{4}), Y_{1T} = 2.2 \sin(T) - 0.8 \sin(\frac{11T}{4})$  for  $T \in [0, 8\pi, \frac{\pi}{12}]$ . (Here a = 1.4, b = 0.8, and the small circle needs to go around the outside of the fixed circle four times to close the curve.)

Sketch both curves here: