MT-A315 Introduction to Linear Algebra Worksheet 5.2

Group: _____ Present: _____

1. Use elementary row operations to find the determinant:

$$|A| = \begin{vmatrix} 1 & 4 & 5 \\ -2 & 3 & 4 \\ 3 & 1 & 9 \end{vmatrix} =$$

2. $$|B| = \begin{vmatrix} 0 & 0 & 0 \\ 2 & 1 & 4 \\ -3 & 10 & 15 \end{vmatrix} =$$

$$|C| = \begin{vmatrix} 7 & 14 \\ 1 & 2 \end{vmatrix} =$$

3. $$|D| = \begin{vmatrix} 2 & 1 & 3 & 0 & 7 \\ 1 & 4 & -1 & 2 & 2 \\ 0 & 3 & 0 & 1 & -1 \\ 1 & 4 & 1 & 0 & -2 \\ 3 & 3 & 2 & 4 & 1 \end{vmatrix}$$ (apply $C_5 \to C_5 + C_4$ and $C_2 \to C_2 - 3C_4$)

$$= \begin{vmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{vmatrix}$$ (now expand along row ____)

$$= (\_\_\_) \begin{vmatrix} & & & \\ & & & \\ & & & \\ & & & \end{vmatrix}_{4\times 4}$$ (apply $C_2 \to C_2 + 2C_1, C_3 \to C_3 + C_1$ and $C_4 \to C_4 - 4C_1$)

(/over)

$= (\_\_\_) \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}$ (now expand along row _____)

$= (\_\_\_) \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}_{3\times 3}$ (apply $C_1 \to C_1 + 5C_3$ and $C_2 \to C_2 + 5C_3$)

$= (\_\_\_) \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}$ (now expand along row _____)

$= (\_\_\_) \begin{vmatrix} & \\ & \end{vmatrix}_{2\times 2}$

$= \_\_\_\_\_$

4. Which of the matrices above are invertible? (Don't compute any inverses) Explain!

5. Let $A$ be an invertible matrix. Prove that $|A^{-1}| = \frac{1}{|A|}$ (Use the product theorem)

Suppose now that $A$ is an orthogonal matrix. What are the possible values for $|A|$? Explain!