Extending partial permutations

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Abstract

Suppose $\triangle_1, \triangle_2, \Gamma_1, \Gamma_2$ are subset of the set of positive integers, $\bar{g} : \triangle_1 \to \Gamma_1$, $\bar{h} : \triangle_2 \to \Gamma_2$ and both $\bar{g}, \bar{h}$ are bijections. The question posed was to find a set Ω of positive integers containing $\triangle_i, \Gamma_i$ for $i = 1, 2$ and $g, h \in \text{Sym } \Omega$ such that $\langle g, h \rangle$, the subgroup generated by g and h, is solvable and $g\big|_{\triangle_1} = \bar{g}$, $h\big|_{\triangle_2} = \bar{h}$.

The problem has its roots in complexity theory from theoretical computer science. The problem may be one that challenges the borders of NP-completeness. Upon my arrival in Tübingen, I studied the test case: $\bar{g} : \begin{matrix} 3 \to 2 \\ 2 \to 1 \\ 4 \to 4 \\ 5 \to 5 \end{matrix}$ and $\bar{h} : \begin{matrix} 1 \to 1 \\ 2 \to 2 \\ 3 \to 4 \\ 4 \to 5 \end{matrix}$.

The talk will focus on two approaches—neither has yet been effective for a solution, but the ideas seemed to have potential.

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