

On the groups of cube-free order

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ABSTRACT

The construction of all isomorphism types of finite groups is an interesting and active problem in group theory. One approach is to classify the groups by order. For example, Hölder examined the groups of square-free order in 1895. We investigated groups of cube-free order; i.e. no third power of a prime divides the order. It is shown that a cube-free group is either solvable or a direct product of a non-abelian simple group with a solvable group. Furthermore, there is a 1–1 correspondence between the solvable cube-free groups of order $n = p_1^{e_1} \cdots p_r^{e_r}$ (prime power factorization) and the solvable Frattini-free groups F with $|F| \mid n$ and $p_1 \cdots p_r \mid |F|$. The proof of this correspondence translates to an efficient algorithm to construct all groups of a given cube-free order up to isomorphism. This algorithm made available the cube-free groups of order at most 50000 in the SmallGroups-Library of GAP.

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