Some finiteness properties in infinite groups

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Abstract

We consider some questions concerning some finiteness properties in infinite groups which are related to Marshall Hall's Theorem. We call these Property S and Property R and both are trivially true in finite groups. To be specific, if A and B are subgroups of a group G, then A and B are said to be *commensurable* if their intersection has finite index in both A and B. A group G satisfies *Property S* if whenever A and B are finitely generated commensurable subgroups of G then their intersection has finite index in their join, i.e., the groups generated by both A and B.

From a result of Mal'cev finitely generated nilpotent groups satisfy Property S. The hypotheses of Mal'cev's theorem we call *Property R* and we show that if a group G and its subgroups satisfy Property R, then G also satisfies Property S.

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