On the capability of nontorsion groups of nilpotency class two

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Abstract

A group G is called *capable* if it is the central quotient of another group H. As a special case of a result by Baer, we have that a finitely generated abelian nontorsion group is capable if and only if its torsionfree rank is greater than one.

Using the classification of 2-generator nontorsion groups of class 2, we obtain the following result.

Theorem 1. *Let G be a 2-generator nontorsion group of nilpotency class 2. Then the following conditions are equivalent:*

  (i) *G is capable;*
  (ii) *G/G' is free abelian of rank 2;*
  (iii) *G/G' is capable.*

The question arises if this result can be extended to finitely generated nontorsion groups of class 2, that is reducing the question on their capability to the one on their abelianizations. This question is motivated by the following result.

Theorem 2. *Let G be a nontorsion group of nilpotency class 2 with $G/\mathrm{T}(G)$ infinite cyclic and $\mathrm{T}(G)$, the torsion subgroup of G, of finite exponent. Then G is not capable.*

However, this is not the case. We have constructed a variety of examples of non-torsion groups of nilpotency class 2, capable as well as non-capable, where in both cases the factor commutator group has torsionfree rank exceeding one. Some of our results can be extended, in particular to higher nilpotency class.

This is joint work with Marcin Mazur and Nor Muhainiah Mohd Ali.

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