

Capability of p -groups of class two and prime exponent

Arturo Magidin

ABSTRACT

A group G is *capable* if there exists a group H such that $G \cong H/Z(H)$. We discuss the problem of determining the capable groups among those p -groups that are of class two and prime exponent.

In this setting, Heineken and Nikolova established a necessary condition for groups that further satisfy $Z(G) = [G, G]$, based on the ranks of $[G, G]$ and $G/Z(G)$. On the other hand, Ellis established a sufficient condition: when there exists a generating set $x_1, \dots, x_n$ that project onto a basis for $G/[G, G]$, and such that the nontrivial generators $[x_j, x_i]$, $1 \leq i < j \leq n$, are distinct and form a basis for $[G, G]$, then G is capable.

We will prove a number of results associated with this problem. In particular, we give a sufficient condition which is closer in flavor to the Heineken-Nikolova necessary condition, based on the ranks of $G/[G, G]$ and $[G, G]$; we also show that the condition $Z(G) = [G, G]$ may be dropped in the Heineken-Nikolova result.

University of Louisiana at Lafayette

`magidin@member.ams.org`