## The intersection map of subgroups and certain classes of finite groups

Matthew Ragland

Abstract

$\mathcal{T}$-*groups* are those groups $G$ in which normality is a transitive relation, that is, those groups $G$ for which $H \trianglelefteq K \trianglelefteq G$ always implies $H \trianglelefteq G$. To generalize, one considers $\mathcal{PT}$-groups and $\mathcal{PST}$-groups. We call a subgroup $H$ of a group $G$ *permutable* (*Sylow-permutable*) in $G$ provided $HK = KH$ for each subgroup (Sylow subgroup) $K$ of $G$. That normal subgroups are permutable (Sylow-permutable) is clear from the definition of normality. So, the class of groups in which permutability (Sylow-permutability) is a transitive relation, the so-called $\mathcal{PT}$-*groups* ($\mathcal{PST}$-*groups*), is a natural way to generalize the class of $\mathcal{T}$-groups. To start the talk, a brief overview of some known results concerning $\mathcal{T}$-groups and their generalizations will be given. The remainder of the talk will concern some recently established characterizations of $\mathcal{T}$-groups and their generalizations in connection with the intersection map of subgroups. Consider the following definition. $H \leq G$ is said to be *normal sensitive* if the intersection map $N \to H \cap N$ sends the lattice of normal subgroups of $G$ onto the lattice of normal subgroups of $H$. A seemingly forgotten theorem of S. Bauman states the following:

**Theorem.** *Let $G$ be a finite group. Every subgroup $H \leq G$ is normal sensitive if and only if $G$ is a solvable $\mathcal{T}$-group.*

Does this result extend nicely to $\mathcal{PT}$-groups as well as $\mathcal{PST}$-groups? Also, are there local characterizations of $\mathcal{T}$-groups, $\mathcal{PT}$-groups, and $\mathcal{PST}$-groups in terms of the intersection map of subgroups? We will answer these questions among others. This is joint work with Jim Beidleman.

*Auburn University Montgomery*
mragland@mail.aum.edu