On simple groups as the union of proper subgroups

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Abstract

Bernhard Neumann showed that a group is the union of finitely many proper subgroups if and only if it has a finite noncyclic homomorphic image. J. H. E. Cohn defined $\sigma(G)$ to be the smallest integer n such that the group G is the set-theoretic union of n proper subgroups. It is well known that there is no group with $\sigma(G) = 2$. The question arises what integers n can occur as $\sigma(G)$ for a group G. By a result of M. J. Tomkinson, $\sigma(G)$ is always congruent to 1 modulo a prime power in case G is solvable, and there is no group with $\sigma(G) = 7$. Cohn showed that $\sigma(A_5) = 10$ and $\sigma(S_5) = 16$. Tomkinson conjectured that there are no groups with $\sigma(G) = 11$, 13 or 15, respectively.

With the help of GAP we determine $\sigma(G)$ for nonsolvable and simple groups in particular. We found that $\sigma(\mathrm{PSL}(2, 7)) = 15$ and that there are no nonabelian finite simple groups with $\sigma(G) = 11$ or 13, respectively. Current evidence supports Tomkinson's conjecture that there are no groups at all with $\sigma(G) = 11$ or 13.

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