The group ring submodule problem

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Abstract

Fix a prime p. Let E be the elementary abelian group of order p^m for some integer $m \ge 1$, and let C be the cyclic group of order p^e for some integer $e \ge 1$. Let P denote the regular wreath product of C by E. Thus the group P has a normal subgroup B that is the direct product of |E| copies of the group C, and the factor group P/B is isomorphic to E and regularly permutes the direct factors in B.

In the context of my research seeking to classify up to isomorphism the monolithic subgroups of certain finite p-groups defined as iterated wreath products, it would be useful to have a full description and list of all the normal subgroups of P that are contained in B.

This problem is equivalent to enumerating the submodules of the regular module of the group ring RE where R is the ring of integers modulo p^e . I have created (and continue to develop) a methodology for approaching this extensive computational problem in an organized and relatively efficient manner. This methodology uses combinatorial ideas, and has enabled me (and some of my students) to solve this problem in some "small" special cases.

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