Classification of locally dihedral amalgams

Abdul Sami

Abstract

An amalgam of rank 2 is a triple \mathcal{A} of finite groups A_1, A_{12}, A_2 such that $A_1 \cap A_2 = A_{12}$. The degree of \mathcal{A} is the pair (d_1, d_2) where d_i is the index of A_{12} in A_i for i = 1, 2. Let the degree of \mathcal{A} be (k, 2) where $k \geq 3$ and suppose that only the identity subgroup of A_{12} is normal in both A_1 and A_2 , let $K = \operatorname{Core}_{A_1}(A_{12})$ and suppose that $A_1/K \cong D_{2k}$ is the dihedral group of order 2k. Then under the above conditions \mathcal{A} is called a *locally* D_{2k} amalgam. Such amalgams were classified for k = 3 by Djoković and Miller and partially classified for k = 4 by Djoković. We classify locally D_{2k} amalgams for all $k \geq 3$ and describe them in terms of generators and relations. We find that if k is odd, then the order of A_1 divides $k \cdot 2^4$ and if k is even there is no upper bound on the order of A_1 , but: if $k \in \{2, 10, 14, 22\} \mod 24$ then A_{12} is an elementary abelian 2-group.

Michigan State University samia@msu.edu