Frobenius problem and dead ends in integers

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Abstract

We show that every finitely generated group has a generating set with respect to which dead ends exist.

We then consider the group of integers. Let a and b be positive, relatively prime integers. We show that the following are equivalent: (i) d is a dead end in the (symmetric) Cayley graph of $\mathbb{Z}$ with respect to a and b, (ii) d is a Frobenius value with respect to a and b (it cannot be written as a non-negative or non-positive integer linear combination of a and b), and d is maximal (in the Cayley graph) with respect to this property. For given integers a and b, we explicitly describe all dead ends in $\mathbb{Z}$. Finally, we show that $\mathbb{Z}$ has only finitely many dead ends with respect to any finite symmetric generating set.

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