Forcing a finite group to be abelian

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Abstract

H. Zassenhaus proved that a group G is abelian provided $N_G(A) = C_G(A)$ for all abelian subgroups A of G. In this paper we seek to extend this result by proving that a finite group is abelian if and only if $N_G(C_G(x)) = C_G(C_G(x))$ for all nontrivial elements $x \in G$.

This result is a consequence of the fact that a finite, nonabelian group must contain some $x \in Z(G)$ such that $C_G(C_G(x))$ is properly contained in $N_G(C_G(x))$.

Other "different" conditions are investigated that imply that a finite group must be abelian.

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