An algebraic approach to twin buildings

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Abstract

Let X be a set, and let S be a partition of the Cartesian product $X \times X$. Assume that $1_X \in S$ and that, for each element s in S, $s^* := \{(y,z) \mid (z,y) \in s\} \in S$. The set S is then called a *scheme* if, for any three elements p, q, and r in S, there exists a cardinal number a_{pqr} such that, for any two elements y and z in X, $(y,z) \in r$ implies $|yp \cap zq^*| = a_{pqr}$.

For any two elements p and q of a scheme S, we define pq to be the set of all elements s in S such that $1 \leq a_{pqs}$. A subset T of a scheme S is called *closed* if $p^*q \subseteq T$ for any two elements p and q in T.

For each subset R of a scheme S, we define $\langle R \rangle$ to be the intersection of all closed subsets of S which contain R as a subset. An element s in a scheme S is called an *involution* if $|\langle \{s\} \rangle| = 2$.

Let S be an association scheme, let L be a set of involutions of S, and assume that $\langle L \rangle = S$. The scheme S is called a *Coxeter scheme with respect to* L if it is constrained with respect to L and if L satisfies the exchange condition (a word by word translation of the group theoretic exchange condition to scheme theory).

It has been shown in [2, Theorem E] that Coxeter schemes can be identified with buildings in the sense of [1]. Moreover, from [3, Theorem 12.3.4] one knows that finite Coxeter schemes are quotients of thin schemes if they do not contain nontrivial thin elements and if the underlying set of involutions has at least three elements.

In my talk, I will discuss the question whether there exists a class of schemes which can be identified with twin buildings in a similar way as Coxeter schemes can be identified with buildings. If yes, would there be an analogue of the above-mentioned theorem on finite Coxeter schemes?

References

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